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# A New Numerical Method for Calculation of Micro- Stress on Unidirectionally Reinforced Plates with Circular Hole In Case of Extension to a Principal Direction

**S. Darya zadeh**

Department for Engineering Physics, National Technical University, Kharkiv Polytechnic Institute, Kharkiv, Ukraine.

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## Abstract

The aim of the present study was to compute the stress concentration in reinforced composite plate with circular hole with respect to the volume ratio of the component materials in composite. The contour of the circular hole and its dependency on the structure of plates were calculated in order to study the behaviors of macro and micro-stresses. The boundary conditions at a large distance from the hole are pressure, uniformly distributed on the plate. Also this problem is analyzed with the finite element method by package ANSYS. The results demonstrated the macro and micro stress and behavior of the orthotropic plate with a circular hole calculated for two different structures.

## 1. INTRODUCTION

The stress concentration behavior in composite plates with holes is always one of the important issues in solid mechanics. Composite materials which consist of two or more constituent materials are commonly used in advanced structural applications, e.g. in the marine and aerospace industry. This is because of appropriate mechanical properties such as high specific strength and stiffness, low density and high resistance to corrosion. However, the limited understanding of the composite material behavior requires more research. This is further complicated by the fact that the behavior of these materials is dependent on lay-up, loading direction, specimen size and environmental effects such as temperature and moisture.

The design of fiber composites which originated from the principles of micromechanics can be further modified to provide desired performances of composite structures. Holes in composites will create stress concentration and hence will reduce the mechanical properties. The solution of the elastic plate theory by the complex functions allows for the solution of isotropic and anisotropic plates with holes. The use of complex variables was first introduced by Muskhelishvili[1]. The functions used for the problems will satisfy the desired boundary conditions. The purpose of this research was to compute the stress concentration around the hole in a composite plate based on the theory of elasticity of anisotropic materials by the application of complex functions Lekhnitskii and Savin [2-3]. Through this method and the

Correspondences:

**S. Darya zadeh**

Department for engineering physics, National Technical University, Kharkiv Polytechnic Institute, Kharkiv, Ukraine.  
[s\\_daryazadeh@yahoo.com](mailto:s_daryazadeh@yahoo.com)

**G. I. Lvov**

Department for engineering physics, National Technical University, Kharkiv Polytechnic Institute, Kharkiv, Ukraine  
[lvovgi@list.ru](mailto:lvovgi@list.ru)

**P. Daryazadeh**

Department of engineering, Damghan Branch, Islamic Azad University, Damghan, Iran  
[manoochehr2507@yahoo.com](mailto:manoochehr2507@yahoo.com)

boundary conditions of pressure distributed, the stress concentration is calculated around the hole in orthotropic plates. Greszczuk studied the stress concentration factors around the hole in orthotropic plate. In his research, stress was applied in one of the principal material directions. Furthermore, he plotted the circumferential stress around the hole for an isotropic material and several unidirectional composite materials [4]. In the literature (for instance in [5-7]) analytical solution for the calculation of composite plates with holes for various cases of load conditions can be found based on the fundamental works of Lekhnitskii. Experimental results of the tensile strain field around circular hole in a composite plate by Toubal et al. [8] was compared with the predictions of a theoretical model previously developed by Lekhnitskii and a finite element study. For a plate containing a hole that is subjected to uniaxial tension or out-of-plane bending, the sensitivity of the stress and strain concentration factors to plate thickness as well as the Poisson's ratio or moment ratio were done by Yang et al. [9, 10] and Yang [11]. Rhee et al. Moreover, [12, 13] examined extensive experimental and numerical studies. For determining the stress concentration around circular, elliptical holes in composite infinite plate subjected to arbitrary uniaxial and biaxial loading at infinity using finite method obtained in works [14- 18]. The study will be using complex functions [19] to determine the effective elastic coefficients of the plates as presented by Vanin (1961). Subsequently, the stress around a circular hole in an orthotropic fiber-reinforced plate is calculated with respect to the volume ratio of the component materials in composite [20].

The stress concentration was analyzed around a hole in a radially inhomogeneous plate by Mohammadi et al. [21]. Numerical methods for calculating the composite material properties typically involve analysis of a representative volume element and a great number of micromechanical models that have been proposed for predicting various mechanical properties of composite materials [22-30].

Next, the micro-stress is calculated around the hole in the plate. The purpose of this paper is to present computational analysis method of micromechanics of strength of materials and to also demonstrate its applications to various micromechanical problems. In this work, the results of the numerical calculations were presented for micro-stress concentration of the different materials in the orthotropic plates for square and hexagonal structures with respect to the volume ratio of the component materials in the composites.

In this report, the finite element method was used to approximate the different elastic properties of the fiber-reinforced composites. A theoretical method and ANSYS (ANSYS, Ver. [11], [NTU "KH PI" company EMT U, r.Kiev, 2010.]) was used to calculate the results of the numerical stress distribution.

## 2. MATERIALS AND METHODS

### 2.1. Determining the effective elasticity constants in composite plates

The composite plate consists of two phases in the matrix-reinforcement. In this plate, it is assumed that the reinforcement fiber is placed along the axis  $x$  in the plate. The determination of the effective elasticity coefficients of the orthotropic plate by knowledge of the mechanical properties of the matrix and reinforcement by mixed functions was proposed by Vanin, G. In formulas (1), (2),  $E_1, E_2, G_{12}, \nu_{12}$  are the modulus of elasticity (Young's modulus), shear modulus and Poisson's ratio of the plate, respectively.  $E, G, \nu$  are modulus of elasticity, shear modulus and Poisson's ratio of the matrix and reinforcement, respectively. The subscripts  $m$  and  $\alpha$  denote the matrix and the reinforcement, respectively. It is assumed that the location of the main axis fibers is along the  $x$ -axis. The following formulas determine the effective elastic constants of the plate based on the mechanical properties of the constituent phases and the volume ratio of the component materials in the composite [19].

$$\begin{aligned}
 G_{12} &= G_m \frac{1-\xi + (1-\xi).G_m/G_a}{1-\xi + (1+\xi).G_m/G_a}, \quad \nu_{21} = \nu_m - \frac{(\chi_m+1)(\nu_m-\nu_a)\xi}{2-\xi + \chi_m\xi + (1-\xi)(\chi_a-1).G_m/G_a}, \\
 E_1 &= \xi E_a + (1-\xi)E_m + \frac{8.G_m\xi(1-\xi)(\nu_a-\nu_m)}{2-\xi + \chi_m\xi + (1-\xi)(\chi_a-1).G_m/G_a}, \\
 \frac{1}{E_2} &= \frac{(\nu_{21})^2}{E_1} + \frac{1}{8G_m} \left[ \frac{2(1-\xi)(\chi_m-1) + (\chi_a-1)(\chi_m-1+2\xi).G_m/G_a}{2-\xi + \chi_m\xi + (1-\xi)(\chi_b-1).G_m/G_a} + \right. \\
 &\quad \left. 2 \frac{\chi_m(1-\xi) + (1+\xi\chi_m).G_m/G_a}{\chi_m + \xi + (1-\xi).G_m/G_a} \right],
 \end{aligned} \tag{1}$$

Equivalency of:

$$\nu_{12} = \nu_{21} \frac{E_2}{E_1};$$

Where

$$\chi_i = 3 - 4\nu_i \quad (i = a, m). \tag{2}$$

In the above equations, effective elasticity coefficients of the plate were dependent upon the volume ratio of the component (fiber). This ratio is dependent upon the manner of placement and the distance between two reinforcement centers which is given by:

$$\xi = \frac{\pi \cdot a^2}{w_1^2 \cdot b \cdot \sin \alpha}. \tag{3}$$

Where  $a$  is the radius of the reinforcement  $0 < \alpha \leq \frac{\pi}{2}$  and  $b > 0$ .

The longitudinal and transverse distance between two reinforcements is equal to  $W_1$  and  $W_2 = W_1 b e^{i\alpha}$ , respectively. The present work is based on the volumetric occupancy coefficient in square and hexagonal arrangements [19].

The angles are between the two vectors in the square  $\alpha = \frac{\pi}{2}$  and hexagonal  $\alpha = \frac{\pi}{3}$  arrangements for  $b=1$ . In the area of numerical calculations, four types of composite plates were examined including 1-carbon fiber plate with aluminum-iron matrix and high-strength carbon fibers, 2- fiberglass plate with epoxide matrix and glass fibers, 3- epoxide resin matrix and epoxide reinforcement and 4-carbon fiber plate with carbon fiber and epoxide resin matrix. Table 1 classifies the mechanical properties of the matrix and reinforcing fibers of plates with respect to reinforcement to the matrix shear modulus ratio of the composite plate [31].

Table1. Illustrates the mechanical properties of the matrix and reinforcement of the composite plate							
Number's materials	$G_a/G_m$	$E_m(Mpa)$	$G_m(Mpa)$	$\nu_m$	$E_a(Mpa)$	$G_a(Mpa)$	$\nu_a$
1	2.49	100000	38100	0.31	250000	94760	0.32
2	20.6	4200	1500	0.4	74800	31000	0.2
3	28.4	2500	947	0.32	70000	26900	0.3
4	68.48	3500	1320	0.32	235000	90400	0.3

The effective elastic coefficients for a square orthotropic plate are calculated using formulas (1), (2) and (3) for  $w_1 = 2.5a$ ,  $w_1 = 2.5a$ ,  $b=1$  and  $\xi = 0.488$  by Maple. Table2 shows the results.

Table2. Mechanical characteristics of the orthotropic plate when  $w_1 = 2.5a$ ,  $b=1$  and  $\xi = 0.488$

Number's materials	$E_1 (Mpa)$	$E_2 (Mpa)$	$G_{12} (Mpa)$	$\nu_{12}$
1	173500	156900	53790	0.32
2	38380	11190	2800	0.3
3	35420	6350	1780	0.31
4	116400	9220	2540	0.31

## 2.2. Stress distribution in infinite orthotropic plate with one circular hole in case of extension to a principal direction

Consider a composite infinite plate with a circular hole, which was extended by a uniformly distributed pressure  $P$  (per unit area) applied at a large distance from the hole. The polar system coordinate is in the center and point  $o$  and angle is assumed to be on axle  $x$ . (Figure. 1).

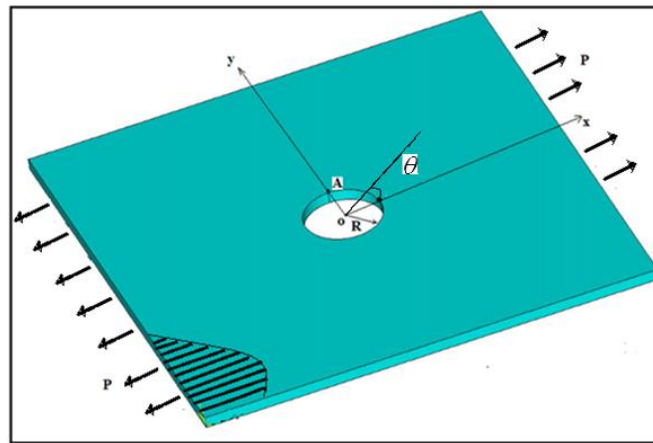


Fig.1. Illustrates the reinforcement plate with a circular hole by an extended pressure  $P$ .

The reinforcing fibers in an orthotropic infinite plate are aligned unidirectional. It is assumed that the main axis of elasticity is the  $x$ -axis. The stress concentration in anisotropic plates with holes was first calculated by Lekhnitskii (1968) using complex functions. Lekhnitskii presented the solution method and related formulas. For ease of use and simplification of formulas, the following relationships are used [2]:

$$m = \frac{E_1}{G_{12}} - 2\nu_{12}, \quad k = \sqrt{\frac{E_1}{E_2}}, \quad n = \sqrt{2k + m},$$

$$\frac{1}{E_\theta} = \frac{\sin^4 \theta}{E_1} + \left( \frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) \sin^2 \theta \cos^2 \theta + \frac{\sin^4 \theta}{E_2}; \quad (4)$$

$E_\theta$  - Young module for tangent (shear) direction.

The parameters  $E_1$  and  $E_2$  is Young's modulus of elasticity for the principal axis.  $\nu_{12}$  and  $G_{12}$  are the Poisson's ratio and the shear modulus on xoy plate respectively, as obtained using the formula (5).

In this document, the problem is studied in linear elasticity mode and the problem is solved for the plate by applying a uniform pressure P on the boundary of plate at a large distance from the circular hole along x- axis. Lekhnitskii provided an expression for circumferential stress around of the circular hole as follows [2]:

$$\sigma_{\theta} = P.f_{(\theta)}; \quad (5)$$

Where

$$\sigma_{\theta} = P \cdot \frac{E_{\theta}}{E_1} \left[ -k \cos^2 \theta + (1+n) \sin^2 \theta \right].$$

$\sigma_{\theta}$  - Circumferential stress and the coordinate  $\theta$  is evaluated based on axle x.

The stress concentration at point A is (Figure. 2):

$$K_1 = \frac{\sigma_{\theta}(Max)}{P}. \quad (6)$$

### 2.3. Micro-stress concentration while applying uniform pressure at a large distance from the hole in an Infinite orthotropic plate

The micro-stress concentration exerted on the plane will be examined. The internal structure of the composite plate was considered and the cells were examined. The arrangements of the cell of the plate on the contour hole at the point where macro stress existed was considered as well. Also, the micro-stress concentrations in the orthotropic plates were calculated, considering the boundary conditions [5, 8-12]. The results were dependent on the internal shape of the plate and the volume ratio of the components, but the analysis was performed by the ANSYS.

### 2.4. Finite element modeling

In this section, the finite element modeling and analysis of a composite plate with a circular hole for unidirectional fibers using ANSYS is discussed. The plate was meshed with PLANE182 elements with four nodes and two degrees of freedom per node in two directions by 10131 elements [4,5,32]. Because of the symmetry for this solution, only the quarter models or cell models were considered and illustrated in Figure 2 [14].

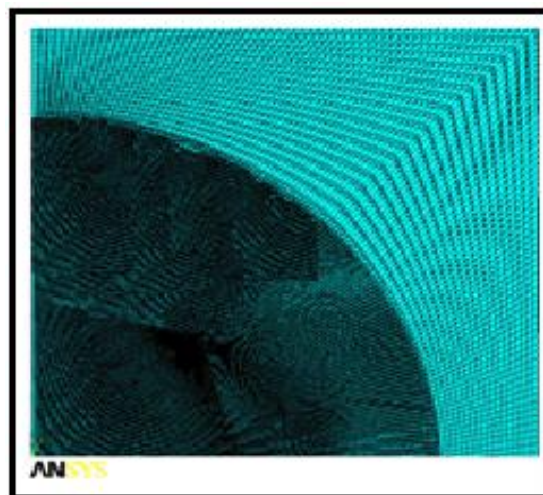


Fig. 2.The quarter model for a representative cell meshed with PLANE2 finite element.

A regular two-dimensional arrangement of fiber in a matrix was adequate to describe the overall behavior of the composite, and was modeled as a regular uniform arrangement. This model assumed that the fiber was a perfect cylinder of radius 0.79, in a square ( $1 \times 1$ ) for the square array and for a hexagonal ( $1 \times \sqrt{3}$ ) of the matrix. It is assumed that the geometry, material and loading of the unit cell are symmetrical with respect to y-z coordinate system as shown in Figure 3. Therefore, 48.8% volume fraction fibers were inserted into the square matrix uniformly as illustrated in Figure 3.

In this case, assuming an element is at point A and a stress,  $\sigma_{\theta(Max)}$  was applied on it, while the change in the arrangement of the element is studied in both square and hexagonal arrangements (Figure. 3). The element is located on the edge of the plate at point A. Following the exerted pressure, stress,  $\sigma_x = \sigma_{\theta(Max)}$ , given the arrangement of the plate, the line  $y=1$  will decrease along the y-axis by  $v_0$ , while  $z=1$  or  $z=\sqrt{3}$  will increase along the z-axis by  $w_0$ .

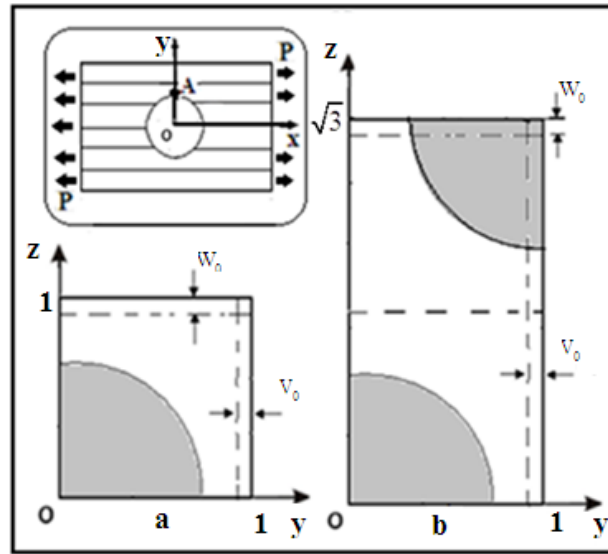


Fig. 3.Representative cell model for: a) square and b) hexagonal fiber arrangement

While applying a uniform parallel pressure to fiber axis, the conditions (7), (8) should be established due to the symmetry and the boundary conditions:

On the lines  $y = 0: v_0 = 0$  and  $z = 0: w_0 = 0$ . In plane  $yo z$ :  $\sigma_x = \sigma_{\theta(Max)}$ .

Subsequently, the boundary conditions are conditions of the periodicity of mechanical fields in view of the deformation:

On the line  $y = 1$  for the square arrangement and hexagonal arrangements:

$$\langle \sigma_y \rangle = \int_0^1 \sigma_y dz = 0, \quad \langle \sigma_y \rangle = \int_0^{\sqrt{3}} \sigma_y dz = 0 \quad (7)$$

On the line  $z = 1, z = \sqrt{3}$  for the square and hexagonal arrangements:

$$\langle \sigma_z \rangle = \int_0^1 \sigma_z dy = 0 \quad (8)$$

The micro-stress concentration is obtained using Eq. (9):

$$K_{(Micro)} = \frac{\sigma_{(Max)}^{Mises}}{\sigma_{\theta(Max)}}. \quad (9)$$

The displacements (10) satisfy the boundary conditions (7), (8) on the cell for a fiberglass plate with square arrangement while applying uniform pressure parallel to fiber axis,  $P=100$ , on the boundary of the hole:

$$\begin{cases} y = 1: v = -0.245e - 8; \\ z = 1: w = -0.245e - 8. \end{cases} \quad (10)$$

### 3. RESULT AND DISCUSSION



In the case of uniaxial tensile unidirectional plate which is parallel to the principal axis, the stress concentration near a hole decreases with an increase in the distance between the centers of reinforcements. It will tend to 3, which indicates the stress concentration in an isotropic plate. Obviously, the stress concentration of the hexagonal arrangement is higher than the square arrangement. Figure 4 shows the stress concentration  $K = \frac{\sigma_\theta}{P}$  around the circular hole versus  $w_l/a$  for  $b=1$  in the case: uniaxial tensile loading along x- axis for a unidirectional fibers plate with a hole by the theoretical method. The four charts for the orthotropic plates composed of the different materials with the properties are listed in Table 2 for both square and hexagonal arrangements.

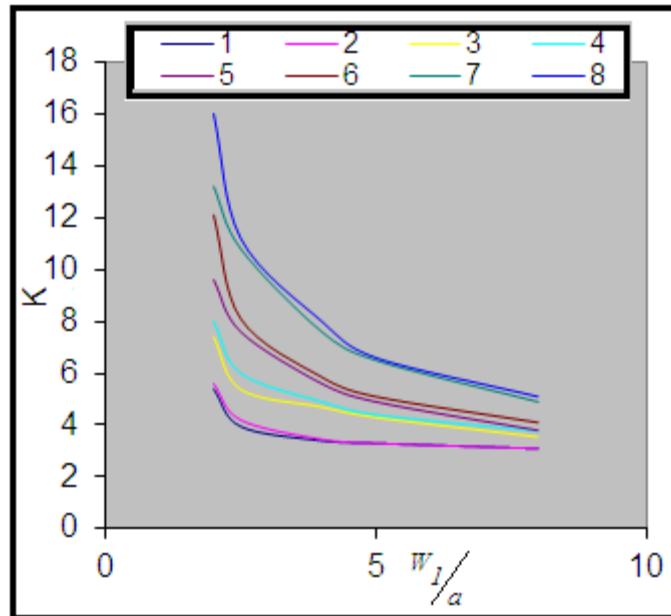


Fig.4. Stress concentration around the hole with respect to the ratios  $w_l/a$  while applying uniform pressure along x- axis for  $b=1$

1, 3, 5 and 7: the charts for materials I, II, III and IV for the square arrangement,  
2, 4, 6 and 8: the charts for materials I, II, III and IV for the hexagonal arrangement.

In this case, Figure 5 shows the calculation of the resultant stress  $\sigma_y$ , and the stress  $\sigma_z$  on a cell using finite element method with the help of ANSYS.

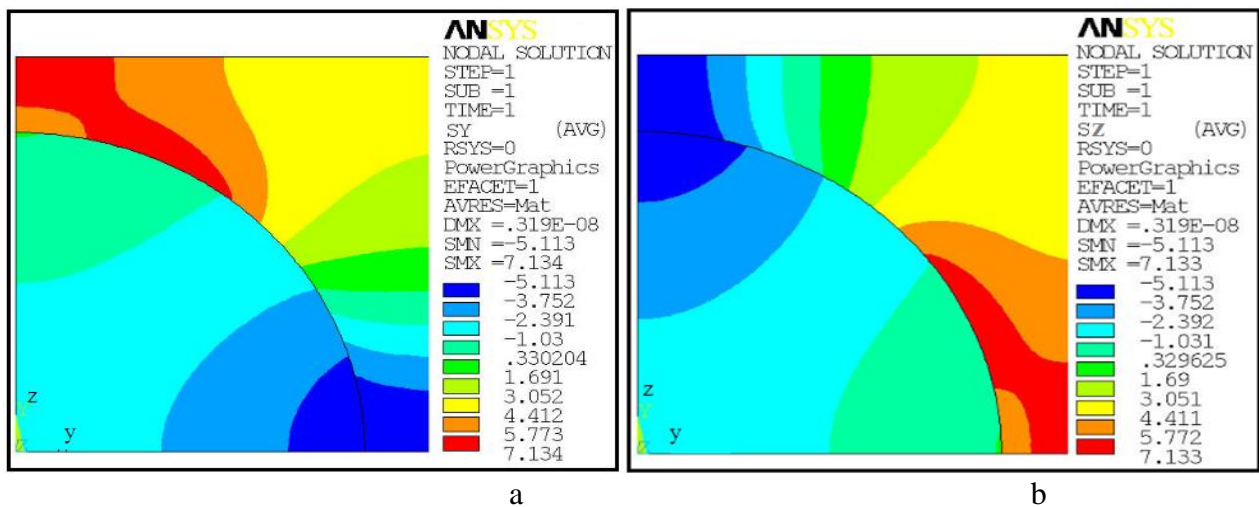


Fig.5. the stress a)  $\sigma_y$  and b)  $\sigma_z$  on the cell consisting of matrix- reinforcement for a fiberglass plate with square arrangement when  $P=100$ .

Figure 6 shows the stress on a cell consisting of the matrix-reinforcement for a fiberglass plate with a square arrangement in the case of uniform stress  $P=100$  distributed along and parallel to the x- axis in the orthotropic plate for  $\xi=0.488$ . The numerical calculations were carried out by ANSYS.

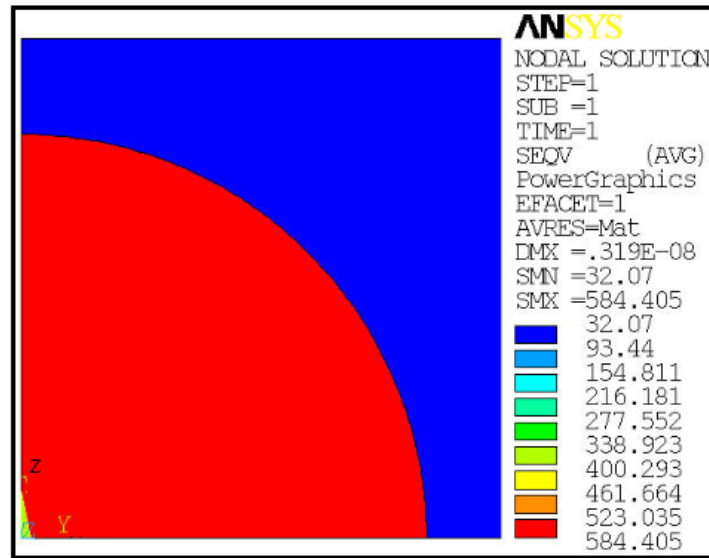


Fig.6. The von Mises stress on the cell consisting of the matrix- reinforcement for a fiberglass plate with a square arrangement while applying uniform pressure,  $P=100$ , along and parallel to the x- axis.

The micro-stress concentration near the circular hole increased from 1 when the distance between the centers of the reinforcements was increased. On the other hand, the micro-stress concentration in the square arrangement is higher than the hexagonal arrangement. The micro-stress concentration around the circular hole versus  $w_1/a$  for  $b=1$  when loading parallel to the principal axis by the finite elements method is shown in Figure 7. The four charts for the orthotropic plates are composed of various materials.

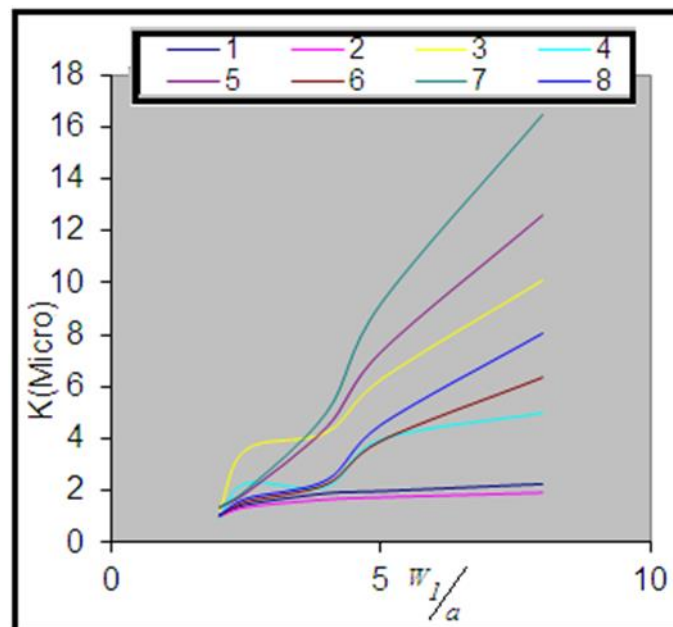


Fig.7. Micro-stress concentration around the circular hole with respect to the ratios  $w_1/a$  while uniform pressure is applied along

x- axis when  $b=1$ ;

1, 3, 5 and 7: the charts for materials I, II, III and IV for the square arrangement,  
2, 4, 6 and 8: the charts for materials I, II, III and IV for the hexagonal arrangement.

The results of this research done on the four plates with the differentiating specifications, confirmed the dependence of the results to the ratio,  $\xi$ , a character of the properties in these constituents. In this research the



proposed method is independent of the complex relationships and mathematical analysis, but dependent on the geometry of the structure.

#### 4. CONCLUSION

In this paper a new approach to the study of stress concentration in composite plates with circular holes is developed. The feature of the proposed approach is to analyze the stress concentration at two levels. The analysis of the concentration of macro-level composite is, however, considered as a homogeneous orthotropic material. Analysis of micro-stress concentration is made at the level of the minimal repetitive structure of the composite. For square and hexagonal fiber reinforcement representative, cell is allocated and boundary conditions are formulated for simulating the stress state of the cell as part of a homogeneous material.

Stress intensity in the most dangerous point is determined by the product of stress concentration factor of micro and macro levels.

Finally, the developed approach makes provision for the determination of the allowable load based on the actual distribution of stresses in thin-walled structural elements which are made of composite materials.

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